Power Law Distribution of Lifespans of Large Firms: Breakdown of Scaling

Paul Ormerod (pormerod@volterra.co.uk)*

Laurence Smith (lsmith@volterra.co.uk)

Volterra Consulting Ltd

The Old Power Station
121 Mortlake High Street
London SW14 8SN

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* Corresponding author

Abstract

Power law distributions of macroscopic observables are ubiquitous in both the natural and social sciences. They are indicative of correlated, cooperative phenomena between groups of interacting agents at the microscopic level. In this paper we argue that when one is considering the survival of the world's largest 100 industrial companies in 1912, the annual rate of extinction does indeed follow power law like behaviour for a significant proportion of the data. The slope of the power law is very similar to that estimated for the extinction rates of biological species.

Crucially, however, there appear to be systematic deviations from this behaviour when one considers the frequency of occurrence of large extinctions. Under these circumstances the power law scaling breaks down. It is argued that it is the adaptive behaviour of the agents which modifies their cooperative behaviour.

1. Introduction

The analytical tools used to characterise the behaviour of complex, disordered systems such as spin glasses [1] has become a paradigm for the study of a wide variety of other systems. These include biological, social and economic systems [2]. In the context of economic and social systems, the behaviour of the system is determined by the interaction of a large number of heterogeneous agents.

Each of these agents has their own objectives (typically the maximization of their individual utilities) and has access only to a restricted subset of all the possible available information (thus each agent makes decisions based upon bounded rationality). The heterogeneous nature of the agents gives rise to a dynamical evolution of the system characterised by interactions which are both frustrated and disordered.

It is commonly assumed that the observation of power-law distributions (fractal behaviour) in a system's macroscopically observable quantities is a characteristic footprint of cooperative behaviour in many-body systems representing the effects of correlated behaviour amongst the constituents of the system. Power law distributions are both self-similar and scale free, demonstrating that events may occur on all length and time scales. It is also argued that the precise details of the microscopic interactions between the systems constituents do not alter the fundamental nature of the distribution of macroscopic events observed i.e. the phenomenon of universality.

There is a crucial difference, however, between ‘dumb’ particles interacting with one another via fixed physical laws and economic agents (individuals) interacting with one another; namely that individuals are capable of adapting their rules of interaction to reflect the economic environment in which they are placed.

For example, in terms of aggregate macroeconomic data (annual growth rates in real per capita GDP in the seventeen leading capitalist economies from 1870 through to 1994), the magnitude and duration of recessions over the business cycle do indeed follow power law like behaviour for a significant proportion of the data. But there are systematic deviations from this behaviour when one considers the frequency of occurrence of large recessions. Under these circumstances the power law scaling breaks down [3].

In this paper, we consider the life spans of the world's largest 100 industrial companies in 1912. By 1995, no fewer than 50 of them had ceased to exist as independent entities. In 53 of the years between 1912 and 1995, none of these firms became extinct, yet in one particular year, 1968, no fewer than six of them did. A power law does provide a very good approximation to this data on the annual rate of extinction, but implies the existence of more years in which relatively large numbers of firms become extinct than has actually been the case.

2. The Data

The years immediately prior to and shortly after 1900 saw a wave of mergers and acquisitions amongst large firms in the West [4], but by 1912 this had settled down, and many of the large firms recognisable today, such as General Electric and Procter and Gamble, had come into existence. The data on large firms in 1912 is analysed in [4], and the world's largest 100 industrial companies by market capitalisation are identified.

Data is not provided on the dates at which firms ceased to be one of the top 100 industrial companies in the world, but it is given for when firms stopped operating as independent agents. This latter phenomenon could arise for a number of reasons, such as bankruptcy, liquidation, acquisition by another company, or nationalisation (as, for example, in Russia in 1917 and France in 1945).

The frequency of annual rates of extinction varies substantially. In most years, no single giant firm became extinct, but four firms became extinct in the year 1919, and no fewer than six in 1968. Figure 1 plots the frequencies.

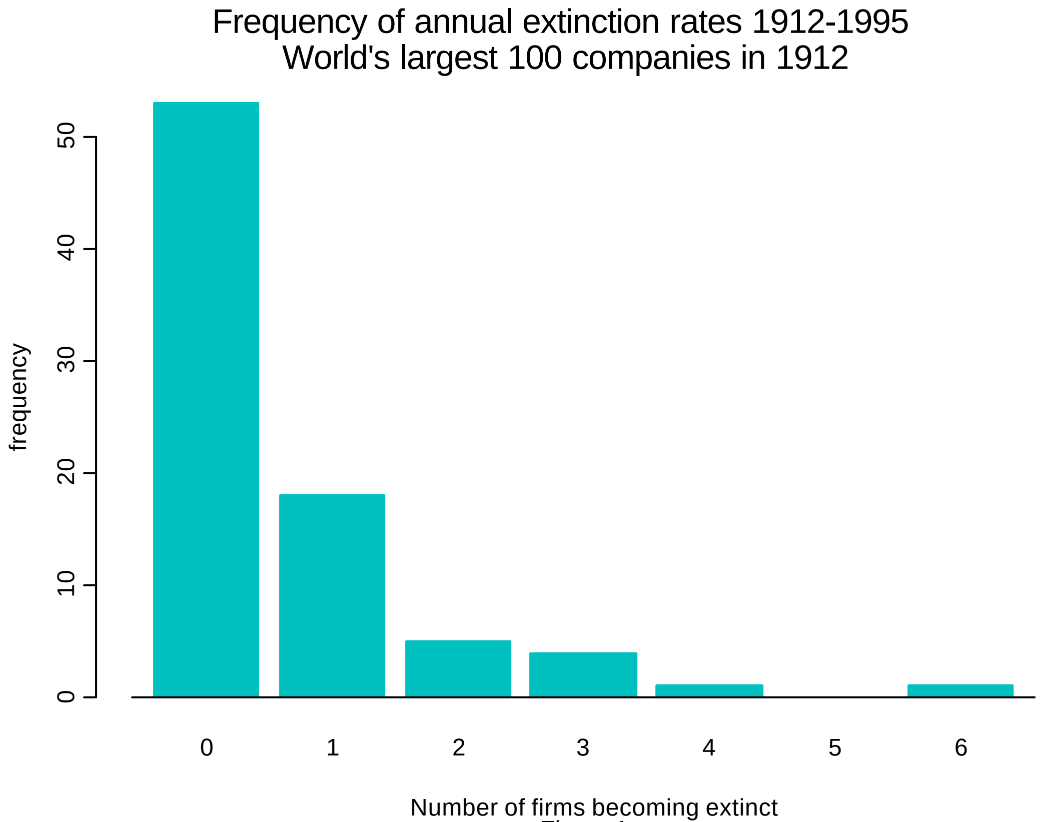

Figure 1

3. The results

A power law of the form

$$F = \frac{a}{N^{b}} \qquad (1)$$

fits the data well, where F is the frequency with which the annual number of extinctions is observed over the 1912-1995 period, and N is the annual number of extinctions.

A non-linear least squares fit of (1) to the data gives estimated values of α of 53.4 and of β of -1.85, the latter with a standard error of 0.14. The standard error of the equation is 2.06. Comparing this latter to the standard error of the data, 19.22, the equation fits the data well.

The estimated slope of the power law (1) of large firm extinction, -1.85, is not statistically significantly different from the slope estimated for the extinction of biological species [5,6], suggesting the possibility of the existence of a more general law of the survival of agents.

The actual values and those fitted by (1) are set out in Table 1.

Table 1. Frequency of annual extinction rates: Actual and fitted by (1)

	Number of extinctions						
	0	1	2	3	4	5	6
Frequency							
Actual	53	18	5	4	1	0	1
Power law fit[1]	53	15	7	4	3	2	1

The problem with the power law description of the data is that it implies the existence of too many years in which relatively large numbers of extinctions take place. An alternative hypothesis of the functional form which best fits the data is that of the exponential distribution:

$$F = \alpha \exp(\beta(N-1)) \qquad (2)$$

In terms of the goodness of fit, (2) has an equation standard error of 1.09 compared to the 2.06 of the power law relationship given by (1). However, the exponential fit is not without its problems also, as Table 2 shows

[1] The fitted data is rounded to the nearest whole number

Table 2. Frequency of annual extinction rates: Actual and fitted by (2)

	Number of extinctions						
	0	1	2	3	4	5	6
Frequency							
Actual	53	18	5	4	1	0	1
Exponential fit	53	18	6	2	1	0	0

In contrast to the power law, the exponential implies the existence of too *few* years in which relatively large numbers of extinctions took place. The power law suggests that there should have been 10 years in which three or more firms became extinct, and the exponential suggests only 3. The actual number is of course 6.

Care is required when so few observations exist, but closer inspection of the relationship between actual and fitted values in the power law relationship suggests that two types of mechanism are in operation. The actual and fitted values from (1) are plotted in Figure 2, along with the 45 degree slope line. With a perfect fit, all observations would lie on this line.

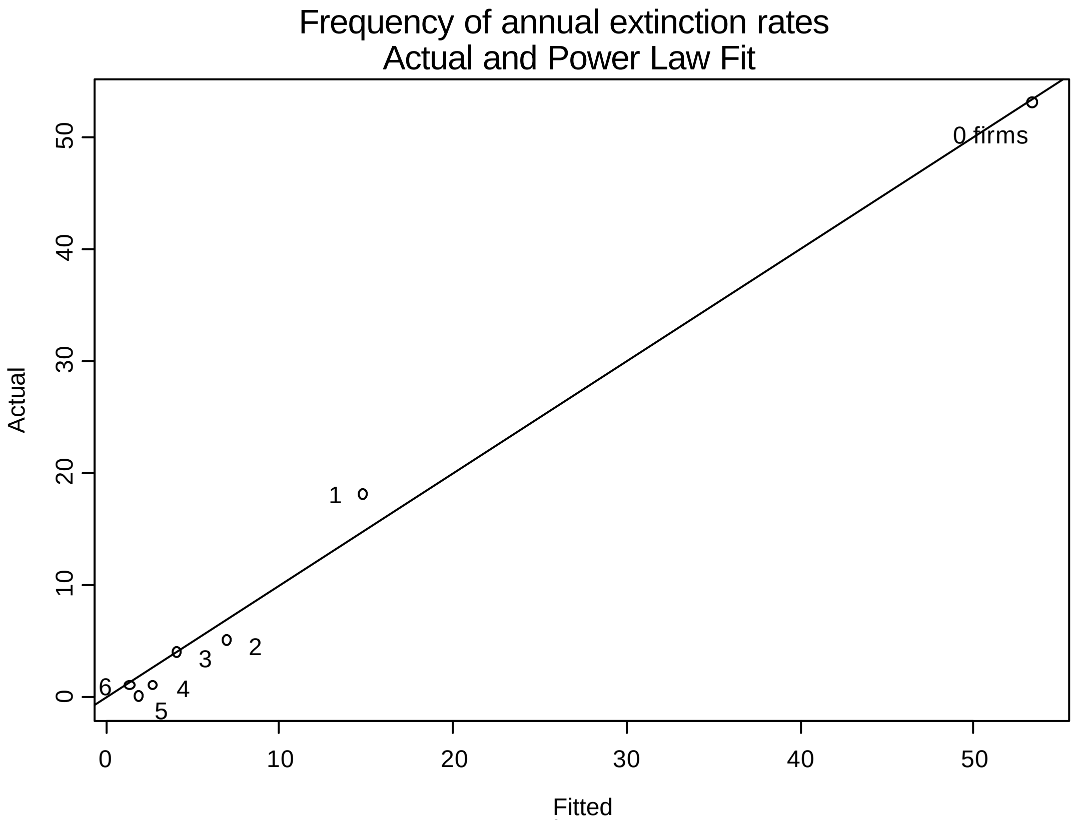

Figure 2

The frequencies with which extinctions were actually observed (ie: from 1 through to 6 firms) appear to lie on a steeper slope than the full data set which includes the frequency with which no firm became extinct.

This is confirmed by a non-linear least squares fit of (1) to the data excluding the frequency zero. The estimated value of β is -2.81, with a standard error of 0.30, and an equation standard error of 0.96 compared to 6.74 of the actual data. The fitted numbers (rounding to one d.p.) of the frequency with which annual extinction rates of four, five and six firms are seen are, 1.4, 0.8 and 0.5, compared to the actual values of 1, 0, 1, which is much more in accord with the actual data than the fit obtained when (1) is fitted to the full data set. In comparison, the fitted values from the exponential (2) over this range are 0.8, 0.3 and 0.1, again implying too few large extinction rates. Estimating a value for the frequency of zero extinctions from the power law equation over the limited sample gives 125, compared to the actual value of 53.

We checked the robustness of these results by grouping the data into observations spanning two years, in other words 1913-14, 1915-16 and so on. The rationale is that shocks which led to the extinction of firms, particularly those involving mergers and acquisitions, might have taken rather more time to feed through than a single year. Qualitatively, the results are identical to those obtained with the basic data itself.

Both a power law and an exponential relationship provide a good fit to the data, with the latter being slightly better than the former. However, the power law over-predicts the number of large extinctions, and the exponential under-predicts them. Further, excluding the observation for the frequency of zero extinctions leads to power law with a higher (absolute) exponent than that obtained with the full data set.

An economic interpretation of this is that the extinction of the world's largest firms is not driven purely by random events of any size. Unlike grains of sand in the sand-pile experiment, large firms have the capacity to resist small shocks. But once an event of sufficient magnitude takes place, extinction does then follow a power law.

4. Conclusion

We have analysed the annual extinction rates of the world's largest 100 industrial companies in 1912 over the period 1912 - 1995. By 1995, no less than 50 of these had become extinct, due to a wide variety of causes such as mergers, bankruptcy and nationalisation. The annual extinction rate varies substantially, with most years showing zero extinctions and one, 1968, showing no less than six.

Both a power law and an exponential distribution provide a close fit to the data. However, the power law over-predicts the number of large extinctions, and the exponential under-predicts them. Care is required in interpreting results with a small number of observations, but the evidence suggests that there are two types of mechanism at work in the extinction of very large firms.

Overall, the evidence suggests that although extinctions of very large companies do have many characteristics of a power law distribution, there is a subtle difference. Once an extinction takes place, a power law mechanism appears to operate in terms of how many firms in total become extinct in the same year. But the observation for the frequency with which no extinctions were observed does not seem to be drawn from the same distribution as the frequencies for extinctions. Unlike grains of sand in the sand pile experiment, large firms appear to have the capacity to resists relatively small shocks.

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